جـامـعـة بنهـــا

كلية الحاسبات والذكاء الإصطناعي

Discrete Mathematics

Lecture 02

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Applications of Propositional Logic (1/13)

1- Translating English Sentences.

2- System Specifications.

3- Boolean Searches.

4- Logic Puzzles.

5- Logic Circuits.

Applications of Propositional Logic (1/13)

1- Translating English Sentences.

2- System Specifications.

3- Boolean Searches.

4- Logic Puzzles.

5- Logic Circuits.

Translating English Sentences

- There are many reasons to translate English sentences into expressions involving propositional variables and logical connectives. In particular, English (and every other human language) is often ambiguous. Translating sentences into compound statements (and other types of logical expressions, which we will introduce later in this chapter) removes the ambiguity.

Applications of Propositional Logic (3/13)

Example 1

You can access the Internet from campus only if you are a computer science major or you are not a student.

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Solution:

Let p, q and r be the propositions:

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

Example 1

(You can access the Internet from campus) **only if** (you are a computer science major or you are not a student).

Solution:

Let p, q and r be the propositions:

$p \rightarrow q$
"p only if q"

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

Example 1

(You can access the Internet from campus) only if (you are a computer science major or you are not a student).

Solution:

Let p, q and r be the propositions:

$p \to q$
"p only if q"

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

The sentence can be represented by logic as $p \to (q \vee \neg r)$

Applications of Propositional Logic (6/13)

Example 2

The automated reply cannot be sent when the file system is full.

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The automated reply cannot be sent when the file system is full.

Solution:

Let p and q be the propositions:

$p \rightarrow q$
"q when p"

p: The automated reply can be sent .
q: The file system is full.

Applications of Propositional Logic (8/13)

Example 2

(The automated reply cannot be sent) **when** (the file system is full.)

Solution:

Let p and q be the propositions:

$p \to q$
"q when p"

p: The automated reply can be sent .
q: The file system is full.

The sentence can be represented by logic as $q \to \neg p$

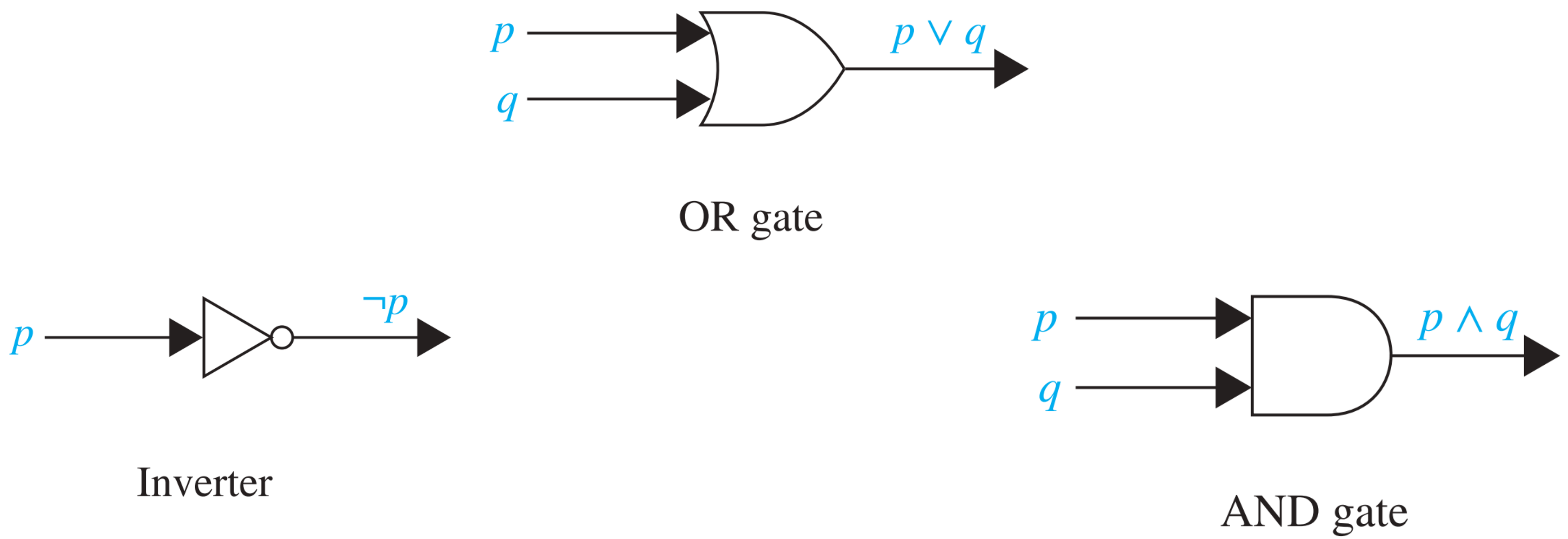
Logic Circuits

- A logic circuit (or digital circuit) receives input signals $p_1, p_2, \ldots, p_n$, each a bit [either 0 (off) or 1 (on)], and produces output signals $s_1, s_2, \ldots, s_n$, each a bit.
- In this course, we will restrict our attention to logic circuits with a single output signal; in general, digital circuits may have multiple outputs.

Applications of Propositional Logic (10/13)

Logic Circuits

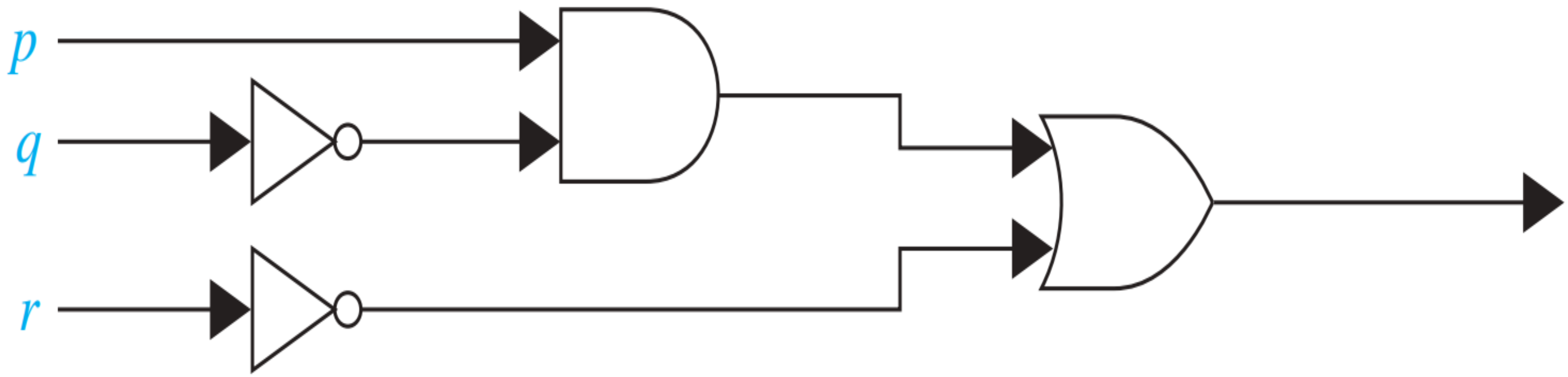
- Complicated digital circuits can be constructed from three basic circuits, called gates.



Applications of Propositional Logic (11/13)

Example 1

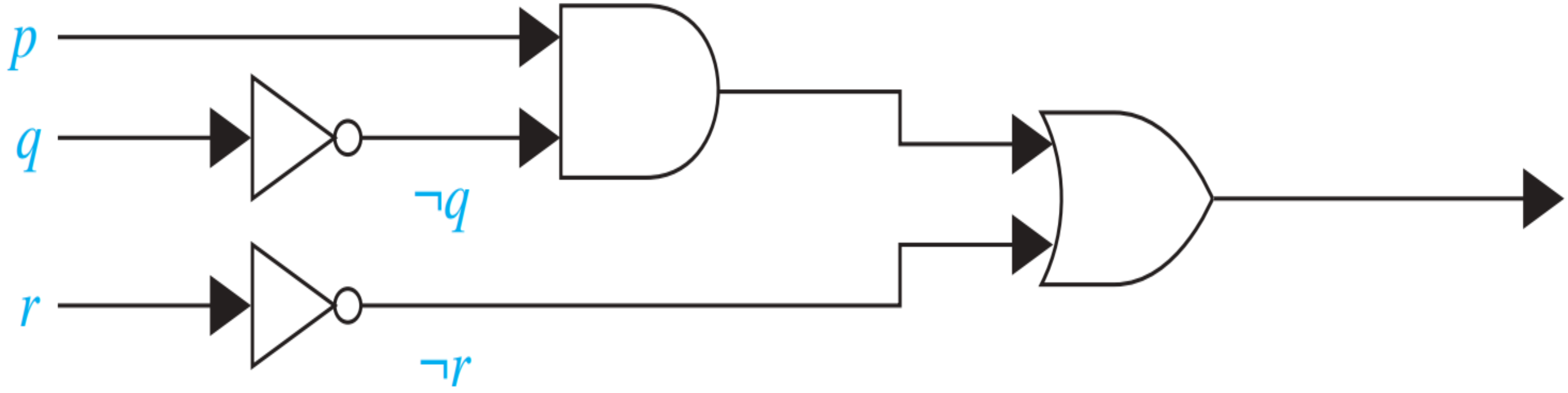
- Determine the output for the combinatorial circuit in the following figure.



Applications of Propositional Logic (11/13)

Example 1

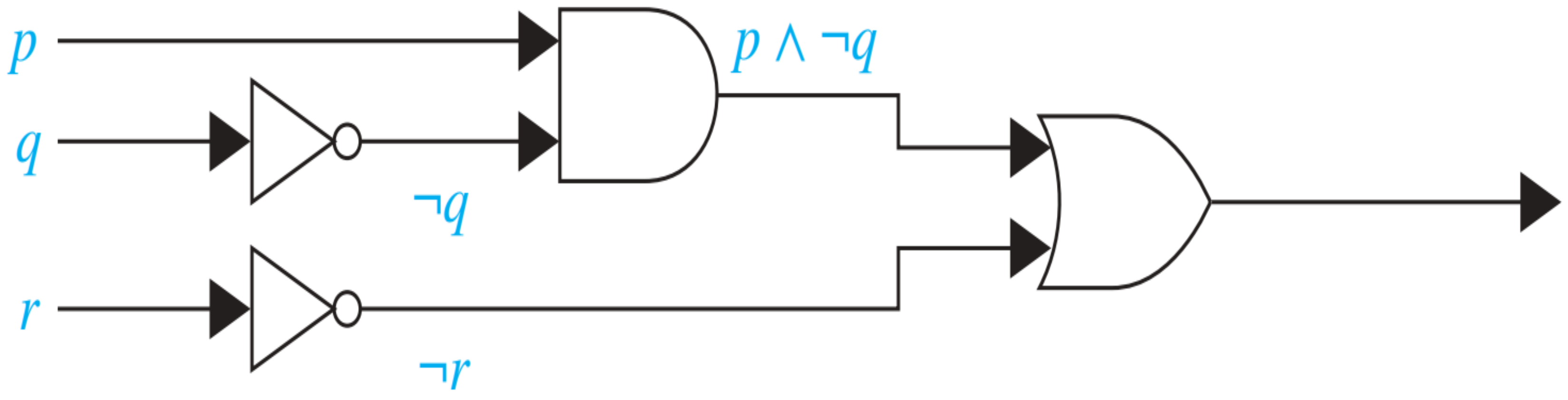
- Determine the output for the combinatorial circuit in the following figure.



Applications of Propositional Logic (11/13)

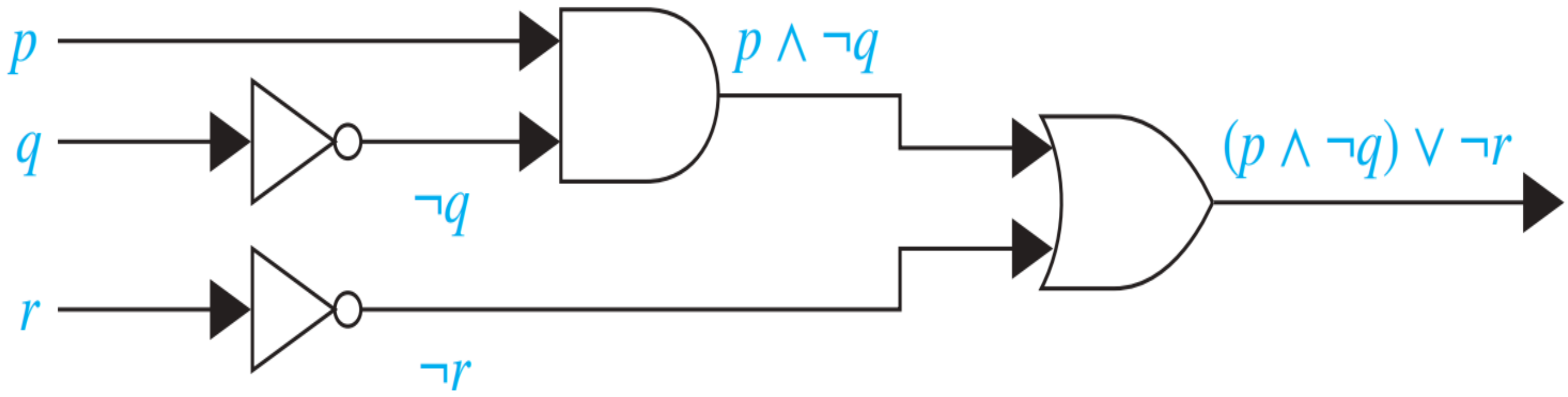
Example 1

- Determine the output for the combinatorial circuit in the following figure.



Example 1

- Determine the output for the combinatorial circuit in the following figure.



Applications of Propositional Logic (12/13)

Example 2

- Build a digital circuit that produces the output

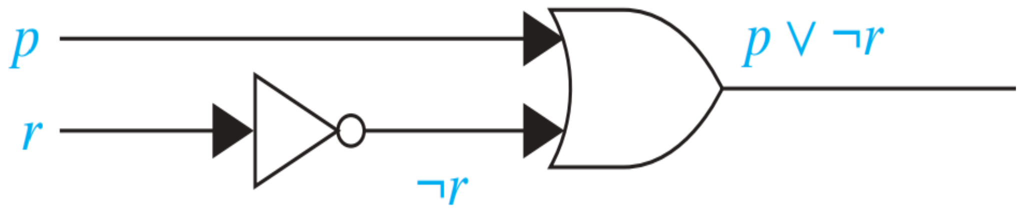
$$(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$$

when given input bits p, q, and r.

Applications of Propositional Logic (13/13)

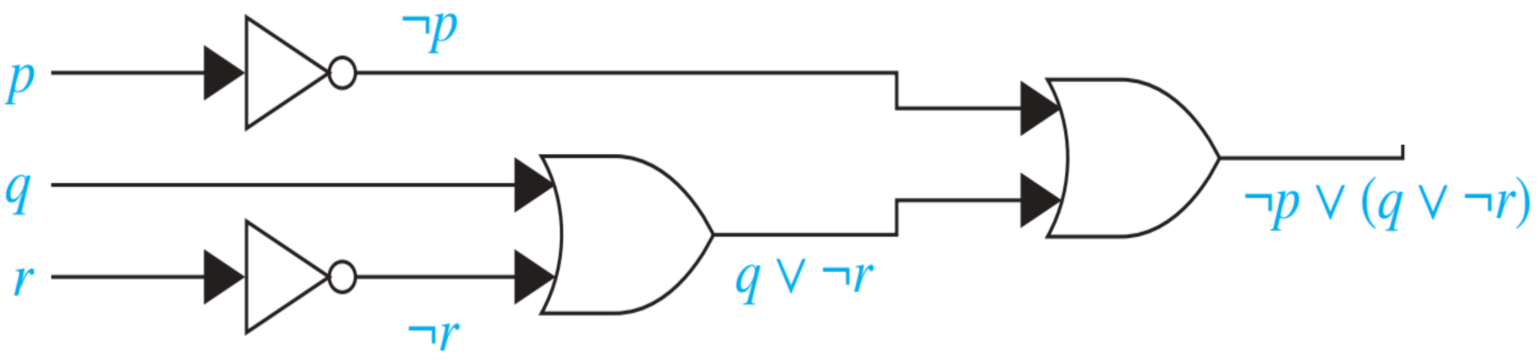
Example 2

$$(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$$



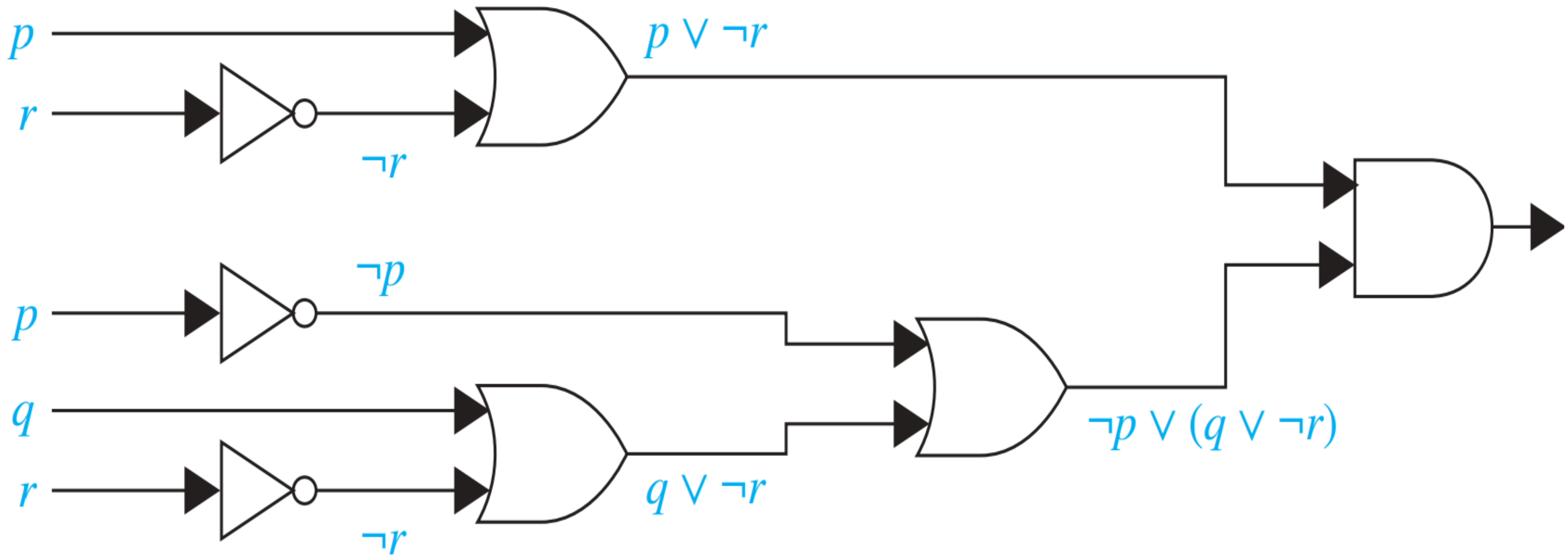
Example 2

$$(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$$

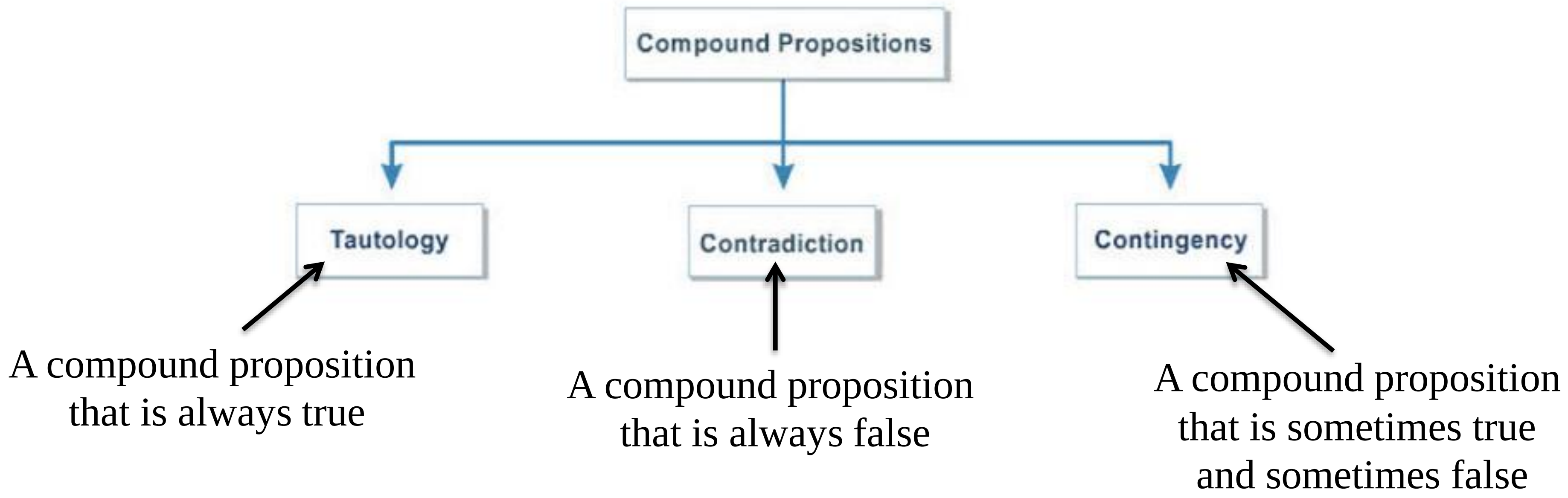


Example 2

$$(p \lor \neg r) \land (\neg p \lor (q \lor \neg r))$$



Compound Propositions Classification (1/2)



Compound Propositions Classification (2/2)

Example:

- Show that following conditional statement is a **tautology** by using truth table

$$(p \land q) \rightarrow p$$

p	q	$p \land q$	$(p \land q) \rightarrow p$

Compound Propositions Classification (2/2)

Example:

- Show that following conditional statement is a **tautology** by using truth table

$$(p \land q) \rightarrow p$$

p	q	$p \land q$	$(p \land q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Logical Equivalences (1/6)

Logically equivalent:

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

Compound propositions that have the **same truth values** in **all** possible cases are called **logically equivalent**.

$$\equiv$$

$$\Leftrightarrow$$

Logical Equivalences (2/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T				
T	F	T				
F	T	T				
F	F	F				

Logical Equivalences (3/6)

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T	F	T	F			
F	T	T	F			
F	F	F	T			

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	
T	F	T	F	F	T	
F	T	T	F	T	F	
F	F	F	T	T	T	

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Logical Equivalences (4/6)

Logical Equivalences (1/3)

Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Logical Equivalences (4/6)

Logical Equivalences (2/3)

Logical Equivalences.	
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical Equivalences (4/6)

Logical Equivalences (3/3)

Logical Equivalences Involving Conditional Statements.
$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$

Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Equivalences (5/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ by the second De Morgan law

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ by the second De Morgan law

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$ by the first De Morgan law

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law

$\neg(\neg p) \equiv p$	Double negation law

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$$
\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law} \\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law} \\
&\equiv \neg p \wedge (p \vee \neg q) && \text{by the double negation law} \\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{by the second distributive law}
\end{aligned}
$$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law} \\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law} \\
&\equiv \neg p \wedge (p \vee \neg q) && \text{by the double negation law} \\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{by the second distributive law} \\
&\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && \text{because } \neg p \wedge p \equiv \mathbf{F}
\end{aligned}$$

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \lor (\neg p \land q))$ and $\neg p \land \neg q$ are logically equivalent.

$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg(\neg p \land q)$	by the second De Morgan law
$\equiv \neg p \land [\neg(\neg p) \lor \neg q]$	by the first De Morgan law
$\equiv \neg p \land (p \lor \neg q)$	by the double negation law
$\equiv (\neg p \land p) \lor (\neg p \land \neg q)$	by the second distributive law
$\equiv \mathbf{F} \lor (\neg p \land \neg q)$	because $\neg p \land p \equiv \mathbf{F}$
$\equiv (\neg p \land \neg q) \lor \mathbf{F}$	by the commutative law for disjunction

$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
$\equiv \neg p \wedge \neg q$	by the identity law for $\mathbf{F}$

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
$\equiv \neg p \wedge \neg q$	by the identity law for $\mathbf{F}$

Predicates and Quantifiers (1/14)

Predicate:

x is greater than 3

Predicates and Quantifiers (1/14)

Predicate:

x is greater than 3

The subject — The predicate

We can denote the statement "x *is greater than* 3" by $\boldsymbol{P(x)}$

where $\boldsymbol{P}$ denotes the predicate "*is greater than* 3" and $\boldsymbol{x}$ is the variable.

The statement $\boldsymbol{P(x)}$ is also said to be the value of the **propositional function** $\boldsymbol{P}$ at $\boldsymbol{x}$. Once a value has been assigned to the variable $\boldsymbol{x}$, the statement $\boldsymbol{P(x)}$ becomes a proposition and has a truth value.

Example1:

Let $P(x)$ denote the statement "$x > 3$."

What are the truth values of $P(4)$ and $P(2)$?

Solution

We obtain the statement *P(4)* by setting $x = 4$ in the statement "$x > 3$." Hence, *P(4)*, which is the statement "$4 > 3$," is true. However, *P(2)*, which is the statement "$2 > 3$," is false.

Predicates and Quantifiers (2/14)

Example1:

Let $P(x)$ denote the statement "$x > 3$."

What are the truth values of $P(4)$ and $P(2)$?

T F

Predicates and Quantifiers (3/14)

Example2:

Let $Q(x, y)$ denote the statement "$x = y + 3$."

What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Predicates and Quantifiers (3/14)

Example2:

Let $Q(x, y)$ denote the statement "$x = y + 3$."

What are the truth values of the propositions

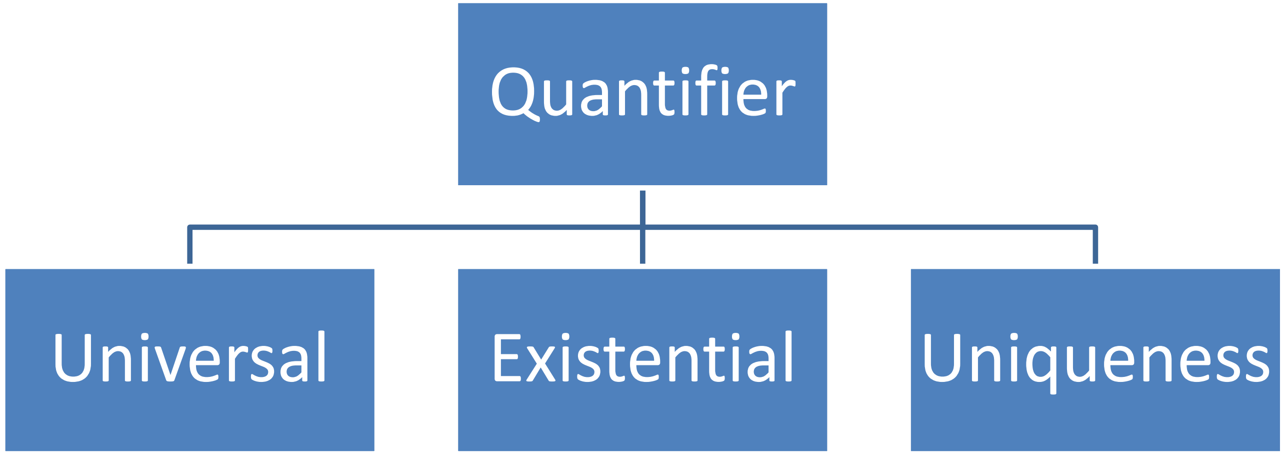
$Q(1, 2)$ and $Q(3, 0)$?

F T

Predicates and Quantifiers (4/14)

Quantifiers:

Expresses the extent to which a predicate is true over a **range** of elements.



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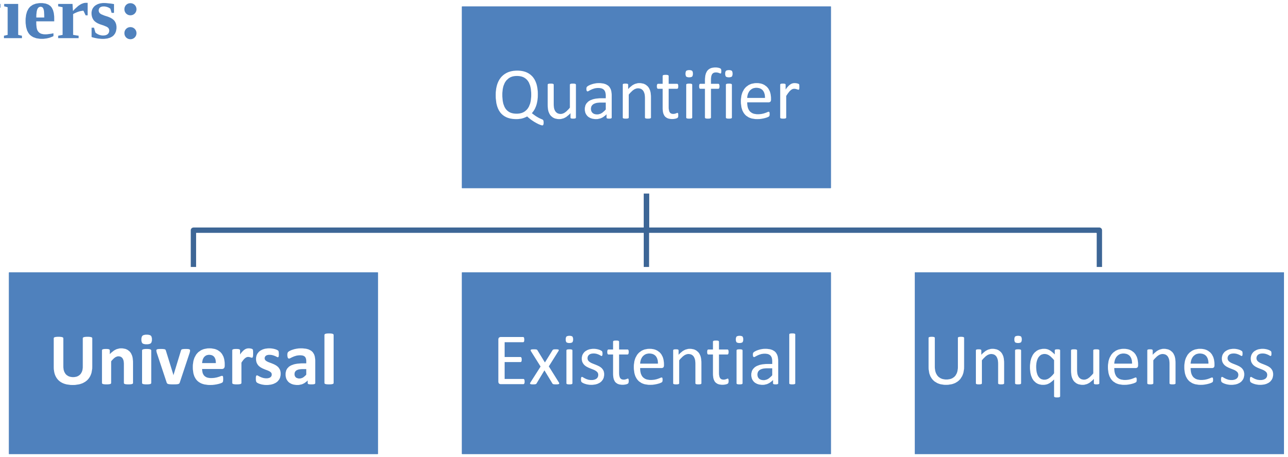
Quantifiers:



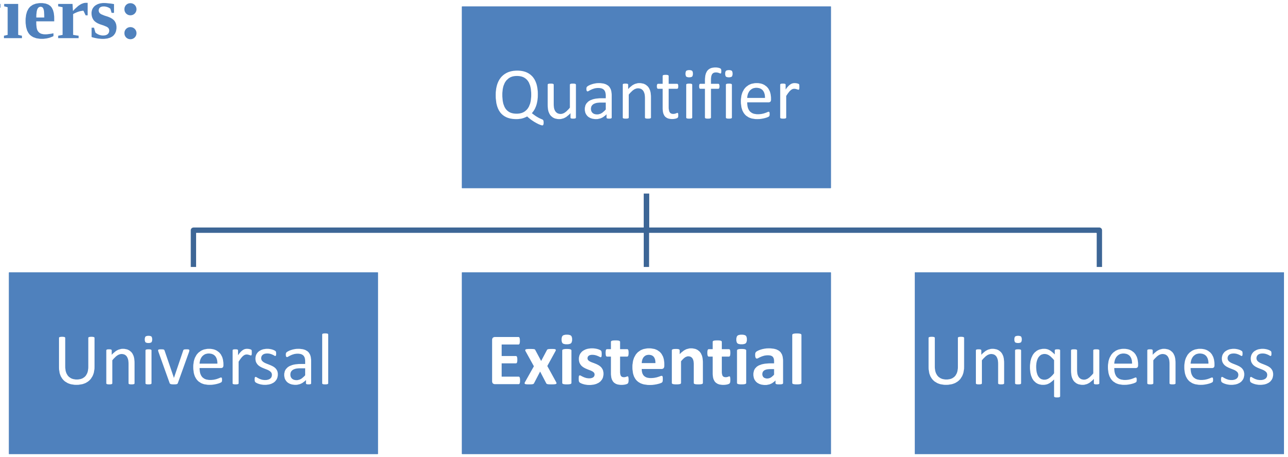
$\forall x\, P(x)$

The *universal quantification* of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain."

Predicates and Quantifiers (6/14)

Quantifiers:



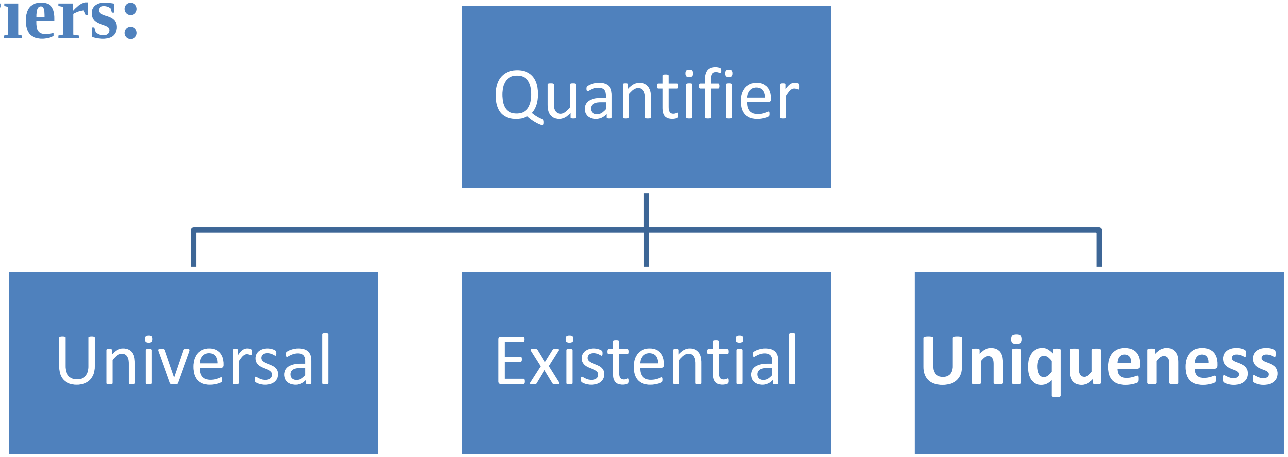
$$\exists x\, P(x)$$

The *existential quantification* of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

Predicates and Quantifiers (7/14)

Quantifiers:



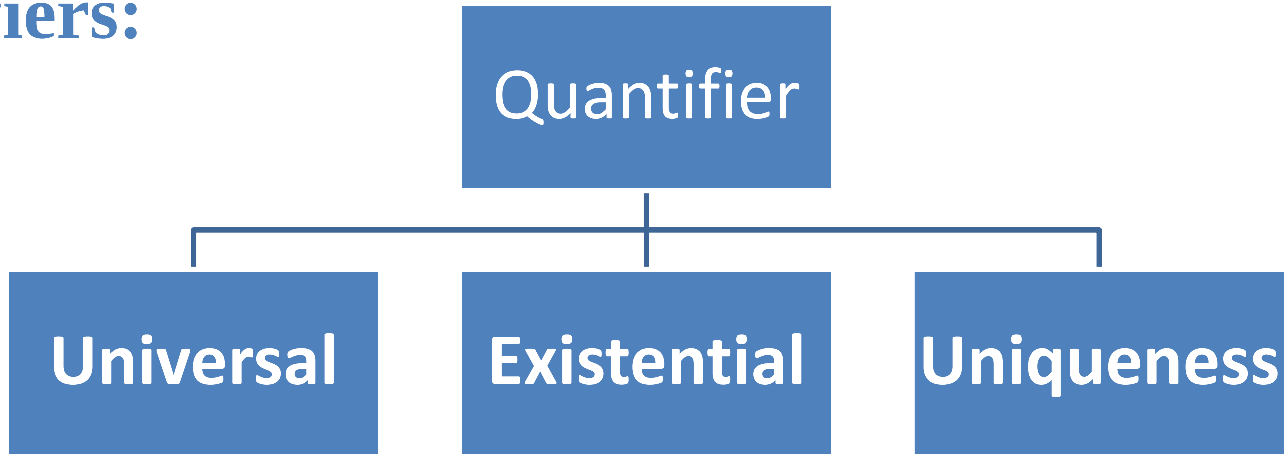
$$\exists!\, xP(x)$$

$$\exists_1 xP(x)$$

"There exists a unique x such that $P(x)$ is true."

Predicates and Quantifiers (8/14)

Quantifiers:



$\forall x\, P(x)$ $\exists x\, P(x)$ $\exists!\, xP(x),\ \exists_1 xP(x)$

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Predicates and Quantifiers (9/14)

Example1:

Express the statement “Every student in this class has studied calculus.

Solution $P(x)$: x has studied calculus.

$S(x)$: x is in this class.

The statement can be expressed as $\forall x(S(x) \rightarrow p(x))$

Predicates and Quantifiers (10/14)

Example2:

Let $P(x)$ be the statement "$x + 1 > x$."

What is the truth value of the quantification $\forall x P(x)$,

where the domain consists of all real numbers?

Predicates and Quantifiers (10/14)

Example2:

Let $P(x)$ be the statement "$x + 1 > x$."

What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?

Solution: Because $P(x)$ is true for all real numbers x, the quantification

$$\forall x P(x)$$

is true.

Predicates and Quantifiers (11/14)

Example3:

Let $Q(x)$ be the statement "$x < 2$."

What is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

Predicates and Quantifiers (11/14)

Example3:

Let $Q(x)$ be the statement "$x < 2$."

What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

Solution: $Q(x)$ is not true for every real number x, because, for instance, $Q(3)$ is false. That is, $x = 3$ is a counterexample for the statement $\forall x Q(x)$. Thus $\forall x Q(x)$ is false.

Predicates and Quantifiers (12/14)

Example4:

Let $P(x)$ denote the statement "$x > 3$."

What is the truth value of the quantification $\exists x P(x)$,

where the domain consists of all real numbers?

Predicates and Quantifiers (12/14)

Example4:

Let $P(x)$ denote the statement "$x > 3$."
What is the truth value of the quantification $\exists x P(x)$,
where the domain consists of all real numbers?

Solution: Because "$x > 3$" is sometimes true—for instance,
when $x = 4$—the existential quantification of $P(x)$,
which is $\exists x P(x)$, is true.

Predicates and Quantifiers (13/14)

Example5:

What is the truth value of $\exists x P(x)$,

where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Predicates and Quantifiers (13/14)

Example5:

What is the truth value of $\exists x P(x)$,

where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Solution: Because the domain is $\{1, 2, 3, 4\}$,

the proposition $\exists x P(x)$ is the same as the disjunction

$P(1) \lor P(2) \lor P(3) \lor P(4)$.

Because $P(4)$, which is the statement "$4^2 > 10$," is true,

it follows that $\exists x P(x)$ is true.

Predicates and Quantifiers (14/14)

Translate into English

Translate the statement $\forall x\left(C(x) \lor \exists y(C(y) \land F(x, y))\right)$ into English, where $C(x)$ is "x has a computer", $F(x, y)$ is "x and y are friends," and both x and y is the set of all students in your school.

Solution

Every student in your school has a computer or has a friend who has a computer.

Negating Quantified Expressions (1/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x \boldsymbol{P}(\boldsymbol{x})$ **:**

Negating Quantified Expressions (1/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

"Every student in your class has taken a course in calculus"

Negating Quantified Expressions (2/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

"Every student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

"Every student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"There is at least one student in your class who has not taken a course in calculus"

Negating Quantified Expressions (2/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$ **:**

"Every student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"There is at least one student in your class who has not taken a course in calculus"

$$\neg \forall x P(x)$$

Negating Quantified Expressions (2/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x P(x)$:

"Every student in your class has taken a course in calculus"

The negation of this statement is

"There is at least one student in your class who has not taken a course in calculus"

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

Negating Quantified Expressions (3/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists x P(x)$:

Negating Quantified Expressions (3/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists x P(x)$:

"At least one student in your class has taken a course in calculus"

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists x P(x)$:

“At least one student in your class has taken a course in calculus"

The negation of this statement is

Negating Quantified Expressions (4/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists x P(x)$ **:**

“At least one student in your class has taken a course in calculus"

The negation of this statement is

"Every student in this class has not taken calculus"

Negating Quantified Expressions (4/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ **:**

“At least one student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"Every student in this class has not taken calculus"

$$\neg \exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$$

Negating Quantified Expressions (4/4)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists x P(x)$ **:**

"At least one student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"Every student in this class has not taken calculus"

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz

Lecture #2: https://www.youtube.com/watch?v=xCDQHgDQEUk&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=4

https://www.youtube.com/watch?v=MipjqNYp3T4&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=5

https://www.youtube.com/watch?v=nYtOiEtcYIs&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=6

https://www.youtube.com/watch?v=mkOkrQZNzoE&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=7

Thank You

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